Lesson 1

IDENTIFY RATIONAL AND IRRATIONAL NUMBERS NY-8.NS.1

INTRODUCTION

Real-World Connection

Elena and Jin are designing a new math app in which users earn points and rewards for correctly solving equations. Because of coding limitations, the app can only include rational numbers in the equations. Elena says an equation can include $\sqrt{2}$ because, like the integer 2, $\sqrt{2}$ is a rational number. Jin disagrees and says that $\sqrt{2}$ is not a rational number because it cannot be written as a fraction. Let's practice the skills in the **Guided Instruction** and **Independent Practice** and help Elena and Jin resolve their dispute at the end of the lesson!

What I Am Going to Learn

- How to determine if a number is rational or irrational

- How to represent rational numbers as fractions

What I May Already Know

- I can write fractions with denominators of 10 or 100 as decimals.

- I know that fractions can represent division of the numerator by the denominator.

- I know what a rational number is and can graph it on a number line.

Vocabulary in Action

A number that can be written in the form $\frac{a}{b}$ is a **rational number**.

- The values a and b are **integers**. The value b is not zero.

- Integers, fractions, and mixed values are rational numbers. For example, 6, -3, 1.8, and $\frac{2}{5}$ are all rational numbers.

- **Terminating decimals** are rational numbers, because they can be expressed as fractions with denominators such as 10, 100, or 1,000.

- For example, 0.4 can be expressed as the fraction $\frac{4}{10}$ or $\frac{2}{5}$, and 2.3 can be expressed as the mixed number $2\frac{3}{10}$.
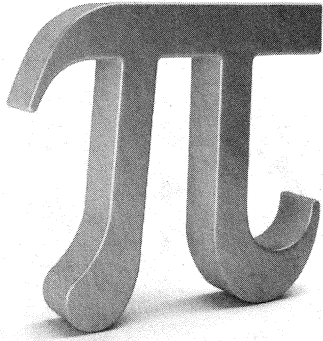
Repeating decimals are rational numbers because they can be expressed as fractions with denominators that are not powers of 10 or factors of 10.

- To show that a decimal is a repeating decimal, write a bar over the portion that repeats. You might recognize $0.\overline{3}$ as $\frac{1}{3}$ or $5.\overline{6}$ as $5\frac{2}{3}$.

An **irrational number** is a real number that cannot be written as a fraction, or ratio, made up of two integers.

- π (pi) is an irrational number because it cannot be written as a fraction. The decimal value of π, 3.14159265…, goes on forever without a repeating pattern.

- Square roots of certain numbers are also irrational numbers. The square root of any prime number is irrational. For example, $\sqrt{2}$ is a decimal that goes on forever without repeating.

- Any number that contains an irrational number, such as $\frac{\sqrt{7}}{4}$ or 2π, is also irrational.

TIPS AND HINTS

Any number that can be written as a fraction is a rational number.

EXAMPLE

Carlos won the election for class president with 125 of the 241 votes. The candidates for president included 3 of the 257 members of the class. Are the integers and fractions involved in this situation rational or irrational numbers?

Step One Remember that all integers are rational numbers.

The numbers 125, 3, and 241 are all integers, so they are all rational numbers.

Step Two Remember that $\frac{a}{b}$ is a rational number if a and b are integers and $b \neq 0$.

Write Carlos's votes as a fraction.

$\frac{125}{241}$

Both 125 and 241 are non-zero integers, so $\frac{125}{241}$ is rational.

Step Three Write the number of classmates who ran for president as a fraction.

$\frac{3}{257}$

Both 3 and 257 are non-zero integers, so $\frac{3}{257}$ is rational.

Step Four Write the fraction of classmates who ran for president and the total number of votes as one fraction.

$\frac{\frac{3}{257}}{241}$

$\frac{3}{257}$ is a fraction, not an integer. Converted to a decimal, $\frac{3}{257}$ is 0.0116731517509728, which is not an integer. Since the numerator cannot be written as an integer, this fraction is irrational.

GUIDED INSTRUCTION

1. Identify the numbers in the chart as rational or irrational.

Number	Rational or Irrational?
$\frac{4}{7}$	Rational
$\sqrt{5}$	Irrational
4.3	Rational
$1.\overline{57}$	Rational

A rational number is a number that can be written in the form $\frac{a}{b}$ where a and b are integers and b does not equal zero: $a = 4$ and $b = 7$, so $\frac{4}{7}$ is rational.
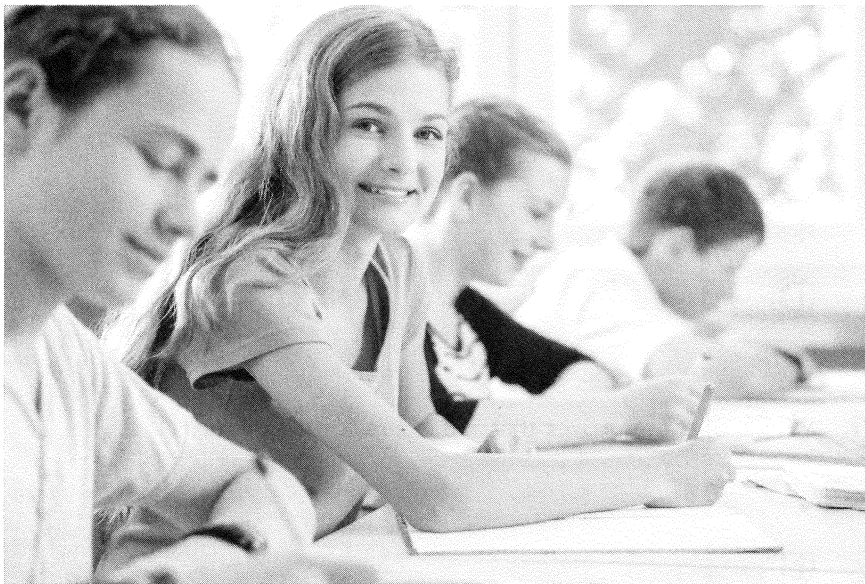
Square roots of prime numbers are irrational, and 5 is a prime number.

Terminating decimals can be written as fractions or as mixed numbers, so they are rational numbers: $4.3 = 4\frac{3}{10}$.

Repeating decimals are rational numbers because they can be written as fractions: $1.\overline{57} = 1.58 = 1\frac{58}{100}$.

◀ **THINK ABOUT IT**

Most numbers you use in your daily life are rational numbers.

2. How can you tell whether $-.3$ and $\sqrt{3}$ are rational numbers?

Step One Begin by seeing if either number involves a terminating or repeating decimal.	**$-.3$ is a terminating decimal.**
Step Two Write the terminating decimal as a fraction with integers for both the numerator and denominator.	$-.3 = -\dfrac{3}{10}$
Step Three Consider whether there is a prime number under the square root sign.	The number 3 is a prime.
Step Four How does the fact that 3 is prime apply here?	The square root of a prime number is irrational.
Step Five Are $-.3$ and $\sqrt{3}$ rational numbers?	**$-.3$ is rational.** **$\sqrt{3}$ is irrational.**

3. Which number is rational?

Ⓐ 4π

Ⓑ $\dfrac{3}{4}$

Ⓒ $\dfrac{\sqrt{5}}{2}$

Ⓓ $\sqrt{11}$

4. Which of these proves that -8 is a rational number?

Ⓐ -0.8

Ⓑ $-\dfrac{8}{1}$

Ⓒ 80

Ⓓ 88

Learning Together

Nikita makes the statement "All non-terminal decimals are irrational numbers." Discuss with a partner. Is Nikita correct? Explain your answer.

Include these ideas in your discussion:

- What are the two types of non-terminal decimals?

- Which type of non-terminal decimal can be represented as $\frac{a}{b}$?

- How are the decimals $4.\overline{3}$ and 4.3564207 alike and different?

- Was Nikita correct?

How Am I Doing?

What questions do you have?

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How can you distinguish between rational and irrational numbers?

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Can you convert from a decimal to a fraction? How?
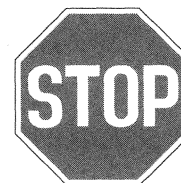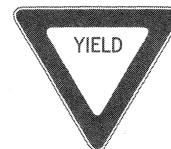
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Circle the sign that shows how you are doing with the skill.

STOP

I am stuck.

YIELD

I almost have it.

I understand the skill.

INDEPENDENT PRACTICE 1

1 Which is an irrational number?

A -1.5

B $\frac{2}{3}$
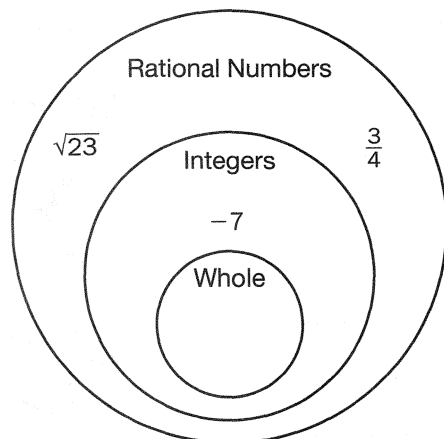
C   3.15151515...

D   6.06006000600006...

2 Which of the following is **not** true of a rational number?

A   It can be a terminating decimal.

B   It can be a repeating decimal.

C   It can be a square root of any whole number.

D   It can be written as a fraction.

3 Sadie placed three numbers on the Venn diagram below.

Did she do it correctly?

A   Yes, they are placed correctly.

B   No, $\sqrt{23}$ is not a rational number.

C   No, -7 is not an integer.

D   No, $\frac{3}{4}$ is not a rational number.

4 Dharma is running for student body representative. He has 100 "Vote for Dharma" buttons that he wants to distribute to students in grades 6, 7, and 8. He divides 100 by 3 to determine how many buttons to distribute to each grade. Explain what will happen when he performs the division. What is the quotient?

Explain your answer.

◀ SKETCH IT

Using small circles to represent the buttons, sketch 10 rows of 10 buttons and use your sketch to check the reasonableness of your answer.

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INDEPENDENT PRACTICE 2

1 Which number is irrational?

A -2

B $3.5\overline{6}$

C π

D $\frac{9}{11}$

2 Which number is irrational?

A $\sqrt{2}$

B $3.\overline{563}$

C 0.4327

D 0

3 Which list shows only rational numbers?

A $-3.4, \pi, 4.\overline{12}, \sqrt{4}$

B $\frac{3}{3}, 4^2, 0.\overline{9}, -45$

C $\frac{\sqrt{7}}{6}, 7.\overline{8}, -8.4, 4\frac{1}{2}$

D $7.5\overline{2}, \sqrt{2}, 8\frac{1}{3}, 0$

4 Which list shows only irrational numbers?

A $29.7, \pi, \frac{77}{99}, \sqrt{1}$

B $\frac{13}{13}, 3^2, 7.\overline{7}, -0.2$

C $12.0\overline{8}, -6, 3\frac{14}{15}, \sqrt{2}$

D $\pi, \sqrt{5}, \frac{\pi}{4}, \sqrt{13}$

5 Ben used the table to describe some numbers.

Number	Real	Rational	Irrational
0.258	•	•	
√29			•
√16	•	•	

How should he fix the table?

A 0.258 should also be an irrational number.

B √29 should also be a real number.

C √29 should also be a rational number.

D √16 is an irrational number.

6 Which number is equivalent to $0.\overline{2}$?

A $\frac{1}{2}$

B 0.22

C $\frac{2}{10}$

D $2.\overline{2}$

7 Which of the following statements must be true?

A If an irrational number is squared, the result is an irrational number.

B The difference between two different irrational numbers is an irrational number.

C The sum of a rational number and an irrational number is a rational number.

D The quotient of two different irrational numbers is a rational number.

8 Alyssa says that $-\frac{\sqrt{13}}{2}$ is a rational number because it is written in the form $\frac{a}{b}$. Is she correct?
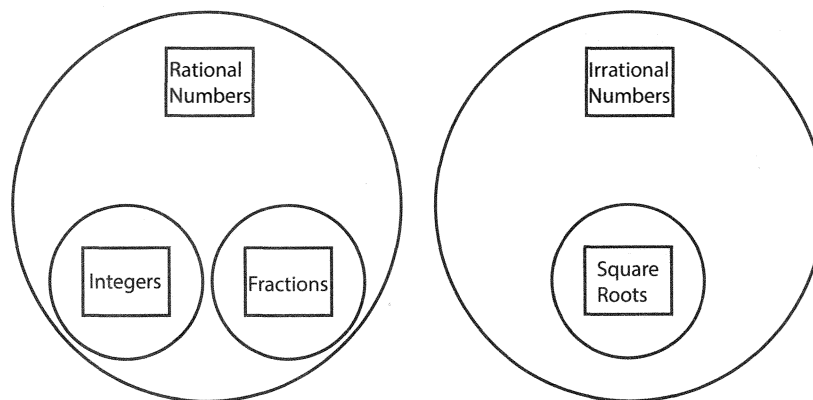
Explain your answer.

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9 This Venn diagram models the relationship between rational numbers, irrational numbers, fractions, square roots of prime numbers, and integers. Explain why you know the diagram is correct.

Explain your answer.

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EXIT TICKET

Now that you have mastered the difference between rational and irrational numbers, let's solve the problem in the **Real-World Connection**.

Elena says an equation can include $\sqrt{2}$ because, like the integer 2, $\sqrt{2}$ is a rational number. Jin disagrees and says that $\sqrt{2}$ is not a rational number because it cannot be written as a fraction. If $\sqrt{2}$ is a rational number, express it as a fraction. If it is irrational, explain why.